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Interest Rate in the Objective Function of the Central Bank and Monetary Policy Design

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Abstract

We analyze two well-known specifications of the interest rate term in the central bank's objective function, and find that the inflation response to a positive demand shock is positive (intuitive) under one specification and negative (counter-intuitive) under the other. We show that the difference between the two responses can be mitigated by a Taylor-type rule and depends on the interest rate inertia. A super-inertial interest rate, which is more aggressive and leads to the counter-intuitive response, may be helpful in an environment of low inflation due to negative demand shocks, such as the current global economic environment.

JEL classification: E58, E61
Keywords: Interest rate smoothing, super-inertial, optimal monetary policy

הכללת גורם הריבית בפונקציית המטרה של הבנק המרכזי ותכנון המדיניות המוניטרית

גיא סגל

תקציר

אנו מנתחים שתי ספציפיקציות מקובלות להכללה של הריבית בפונקציית המטרה של הבנק המרכזי, ומוצאים שתגובת האינפלציה לזעזוע ביקוש חיובי היא חיובית (אינטואיטיבי) תחת ספציפיקציה אחת, ושלילית (לא אינטואיטיבי) תחת הספציפיקציה האחרת. אנו מראים שניתן לשחזר את ההבדל בתגובת האינפלציה באמצעות כלל מורחב מסוג טיילור בכפוף למידת האינרציה של הריבית בכלל. תגובה סופר-אינרטית, המאופיינת באגרסיביות, מובילה לתגובה הלא אינטואיטיבית, ועשויה להיות לעזור בסביבה של אינפלציה נמוכה בעקבות זעזועי ביקוש שליליים, כמו הסביבה הכלכלית העולמית בשנים האחרונות.

1. Introduction

This paper focuses on the implications of modeling an optimal rule/targeting rule, that is, the rule that minimizes the objective (loss) function of the central bank subject to the economy's model. The basic objective function includes the squared deviations of inflation and output gap from their targets.[1] However, in light of the observed tendency of policymakers to smooth interest rates, the commonly used objective function also includes an interest rate term. Some explanations for interest rate smoothing include the central bank's tradeoff between its concern for stability of the financial system and for price stability (Cukierman 1996)[2]; interest rate smoothing in forward-looking models as an anchor for expectations results in lower volatilities of inflation and output (gap) as well as of the interest rate (Levin, et al., 1999, Woodford, 1999, 2001); and also reflects uncertainty regarding data due to revisions (Orphanides 1998). A gradual response may also be in order given uncertainty regarding the parameters of the economy (Sack 2000), the striving for consensus about interest rate decisions (Sack and Wieland 2000), or avoiding the (zero) lower bound (Woodford 2002).

This paper compares and analyzes the implications of each of the two most well-known specifications of the interest rate term in the objective function of the central bank (hereinafter, CB)—i.e., adding squared terms to either the interest rate (hereinafter, OFI) or to its first difference (hereinafter, OFII)—on optimal monetary policy within the timeless perspective framework (Woodford 2003a).

Woodford (2003a) showed that in the presence of a zero interest rate lower bound, or in the presence of non-negligible transaction frictions, the microfounded basic objective function should be augmented with a squared interest rate term (OFI). While Woodford's result does not directly reflect considerations of interest rate smoothing, it does yield a degree of such smoothing. Many central bankers as well as academics used Woodford's derived objective function to address optimal monetary policy issues, such as Billi (2012), Giannoni (2012) and Kam (2003), among numerous

[1] Rotemberg and Woodford (1999) derive the basic objective function analytically within the New Keynesian model using a second-order Taylor approximation of the representative household's utility function.

[2] Cukierman (1996) describes the following mechanism: Given the asymmetry between the long term loan contracts the banks give, most of which are fixed rate, and the short term deposits, banks' profits are negatively related to the short term interest rate. Hence, in order to strengthen the stability of the financial system, the interest rate should be decreased, which in turn leads to inflationary pressures, and vice versa. This tradeoff between price stability and stability of the financial system supports interest rate smoothing.

others. In contrast, other authors—e.g., Adolfson et al. (2011), Dennis (2005), Givens (2012) as well as Woodford (2000)—have augmented the basic objective function in an ad hoc manner with a squared *first difference* of the interest rate (OFII). The Norges Bank uses a combination of both squared *first difference* of the interest rate and squared interest rate in its loss function (Evjen and Kloster 2012).

We show that the optimal policy rule for OFII derived in the paper—that includes the first difference of the interest rate—embodies a less inertial response of the interest rate to its first lag and a more smoothed path, in comparison to its counterpart in OFI, derived by Giannoni (2012). The main difference between the two OFs is in the inflation response to a positive demand shock: it is negative under OFI and positive under OFII. This difference may explain in part why OFII, though not theoretically based as OFI, is more frequently used in the structural models used by central banks.

Following Giannoni (2012), we show that under OFI, the optimal interest rate may be represented by an augmented Taylor rule with two lagged interest rates, in which the sum of the coefficients of the lagged interest rate is greater than one—previously denoted as “super-inertial” (Giannoni (2012) and Rotemberg and Woodford (1999)). While Giannoni (2012) concentrates on OFI, we show that under OFII, the optimal interest rate may be represented by a rule with three lags of the interest rate and one lead, beyond a response to inflation and output gap. However, in this case, the sum of the coefficients of the interest rates is one.

Based on this analysis and assuming an augmented Taylor rule with two lags of the interest rate, using simulations we show that if the sum of the coefficients of the lagged interest rate is greater than one, the inflation converges to steady state from below zero after a positive demand shock, as in OFI. For a sum of coefficients which is less than one, the inflation converges to steady state from above, as in OFII—the standard response. Hence, reversing the impulse response's signs when the sum is greater than one implies that such calibration is helpful in an environment of low inflation due to *negative* demand shocks, such as the current global economic environment—the main contribution of the paper.

This inflation response stems from the “super-inertial” characteristic of the rule, which, we argue, is the opposite of inertial; when the sum of coefficients of the lagged interest rate approaches one from below, it does lead to a smoother interest rate response. However, when the sum is greater than one, it leads to a more aggressive response and less inertia in the interest rate. This aggressiveness leads to a higher real ex-ante interest rate, which lowers both output gap and inflation to below zero, and hence, as noted by Woodford (2003_b) and Giannoni (2012), does not explode. Moreover, the expected higher real ex-ante interest rate offsets the aggressiveness, and hence the observed difference between the interest rate responses, when the sum of the coefficients is either above one or below one, is minor.

The rest of the paper is organized as follows. Section 2 presents the two objective functions with respect to the interest rate. Section 3 shows the optimal policy rule under OFI, and Section 4 derives the optimal policy rule under OFII. Section 5 analyzes and compares the two specifications, Section 6 presents sensitivity analysis, Section 7 proposes an augmented Taylor rule which can be designed to lead to either a positive or a negative inflation response to a positive demand shock, and Section 8 concludes.

2. The Objective Function of the Central Bank

2.1. Objective Function I – squared interest rate term

First, we consider the objective function:

$$E_t\{L_t\} = 0.5E_t \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2\right], \tag{1}$$

where π_t is the inflation rate, x_t is the output gap, and i_t is the nominal interest rate. The operator E_t denotes the mathematical expectation as of time *t*. The first two terms reflect the standard flexible inflation targeting regime. The main goal of this regime is to achieve price stability, captured by the first term, and to stabilize output fluctuations, represented by the squared output gap in the second term. λ_x is the weight of the output gap relative to inflation.

As for the third term, Woodford (2003) showed that the squared interest rate enters the objective function of the central bank (the third term) in the presence of a zero interest rate lower bound (p. 428) or in the presence of non-negligible transaction frictions (p. 476). λ_i is the weight of the interest rate relative to inflation.[3] While this

[3] Woodford (2003, Ch. 6) shows that λ_i is a function of λ_x,—the interest rate semielasticity of money demand and the velocity of money, among other variables, in the case of transaction frictions—or that it is greater when accounting for the zero interest rate lower bound.

objective function is not derived from an interest rate smoothing objective, it does imply a degree of such smoothing.

Figure 1 depicts monetary interest rates of selected central banks during the January 1990–December 2016 period. The figure shows the observed tendency of central banks to smooth interest rates: an interest rate change is in general followed by further changes in the same direction.

Central Banks' Interest Rates

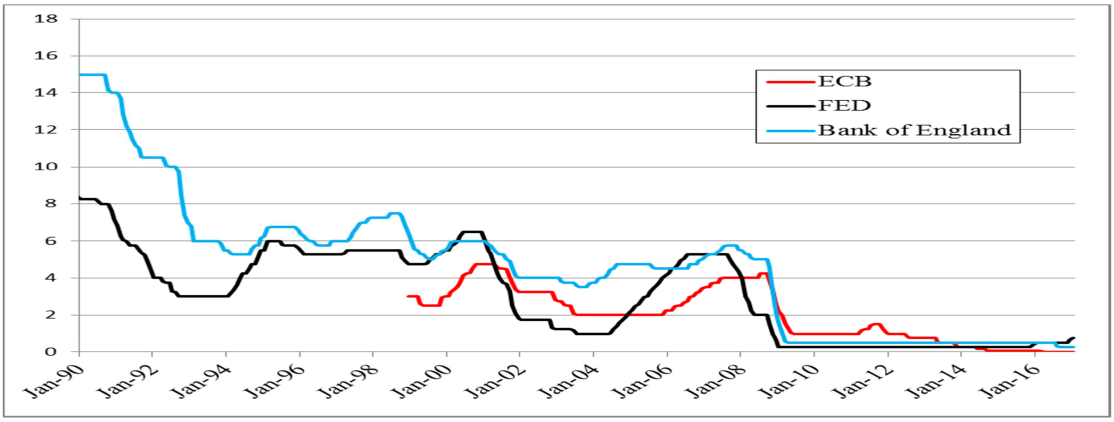

Fig. 1. Central Banks' Interest Rates, 1990–2016.
Source: Bloomberg.

2.2. Objective Function II – squared first difference of the interest rate

The objective function of the central bank in this case is:

$$E_t\{L_t\} = 0.5E_t \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_x x_t^2 + \lambda_{\Delta i}(\Delta i_t)_t^2\right]. \tag{2}$$

The third term in this objective function, $\Delta i_t = i_t - i_{t-1}$, is an *ad hoc* supplement, which is designed to yield interest rate smoothing.

We address the implications of using each of the above two specifications of the objective function on the economy by computing the timeless perspective optimal policies within the canonical New Keynesian (NK) model:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t , \tag{3}$$

$$x_t = E_t\{x_{t+1}\} - \sigma(i_t - E_t\{\pi_{t+1}\}) + g_t . \tag{4}$$

Equation (3) describes the inflation dynamics known as the New Keynesian Phillips curve (NKPC). The dynamic IS equation (4) describes the output gap dynamics. The disturbance term g_t denotes white-noise demand shock, and may reflect preference shocks or shocks to the natural interest rate.[4] Specifically, we use Woodford's (2003) specification for the structural parameters κ and λ_x:

$$\kappa = (1-\theta)(1-\beta\theta)(\omega+\sigma^{-1})/[\theta(1+\omega\varepsilon)], \quad (5)$$

$$\lambda_x = \kappa/\varepsilon. \quad (6)$$

θ is the Calvo parameter; (1- θ) is the probability of a firm resetting its prices. ω is the elasticity of real marginal cost with respect to production (Woodford (2003) p. 148–152). σ is the intertemporal elasticity of substitution in consumption, and ε is the elasticity of substitution among goods. In the baseline calibration (Appendix C) we set the ratio of λ_i/λ_x following Woodford (2003).

3. The Optimal Interest Rate Rule under the OFI specification

Under OFI, Giannoni (2012) derives the optimal interest rate (Appendix A):

$$i_t = (\sigma\kappa/\lambda_i)\pi_t + (\sigma\lambda_x/\lambda_i)\Delta x_t + (1+\sigma\kappa/\beta)i_{t-1} + \beta^{-1}\Delta i_{t-1}\ . \quad (7)$$

The main difference between this policy rule and the rule which is derived from the basic objective function without an interest rate term (Appendix A.1) is that here the interest rate responds to the *change* in the output gap and to the first and second lagged interest rates. Another main difference is that in the basic objective function, policy sterilizes the demand shock, as there is no tradeoff between stabilizing inflation and the output gap and stabilizing the interest rate. Giannoni notes that the optimal rule in Equation (7) implies a “*super-inertial*” response of the interest rate, reflected in the sum of coefficients of the lagged interest rate $1+\sigma\kappa/\beta$ (Woodford 2003, Giannoni 2007 & 2012). This “super-inertial” response is a desirable character of monetary policy in models with forward-looking agents when the objective function also includes the interest rate, as it anchors expectations, and by doing so makes it possible to achieve lower fluctuations of inflation and output gaps with minor policy (interest rate) actions (Rotemberg and Woodford, 1999, and Woodford 2003b). Amato and Laubach (2003) show that super-inertial policy remains optimal in a new Keynesian model with inflation expectations and habit formation. However, as we

[4] Cost-push shocks, i.e., shocks to the Phillips curve, affect the economy in a similar way under the tested objective functions in this paper, and hence we focus on the demand shocks.

show below, the "super-inertial" characteristic is less inertial than in a case when the coefficient of the lagged interest rate is less than one.

4. Derivation of the interest rate rule under the OFII specification

The first-order condition with respect to the interest rate under the OFII specification, Equation (2), is given by:

$$\partial W_t / \partial i_t = \lambda_{\Delta i} \Delta i_t - \beta \lambda_{\Delta i} E_t \{\Delta i_{t+1}\} + \sigma \varphi_{1,t} = 0 \quad \forall t, \tag{8}$$

while the rest of the first-order conditions are the same as under OFI (Appendix A). The first-order conditions together with the behavioral equations now yield the interest rate rule (Appendix B):

$$\begin{aligned} i_t &= [(\sigma\kappa)/(\lambda_{\Delta i}\Omega)]\pi_t + [(\sigma\lambda_x)/(\lambda_{\Delta i}\Omega)]\Delta x_t + (\beta/\Omega)E_t\{i_{t+1}\} + (1-\beta/\Omega)i_{t-1} \\ &\quad + [(\Omega-1)/(\beta\Omega)]\Delta i_{t-1} - (\beta\Omega)^{-1}\Delta i_{t-2}, \end{aligned} \tag{9}$$

where $\Omega \equiv \sigma\kappa + 2(1+\beta)$.

Note that in the optimal interest rate rule under OFII, the sum of the coefficients of the interest rate terms is one, while in the optimal interest rate rule under OFI—Equation (7)—it is above one, $1 + \sigma\kappa/\beta$. The trade-offs among the target variables are reflected in both optimal interest rate rules. A higher weight for the interest rate term in the objective function yields a lower response to inflation and to the output gap; a higher weight for the output gap increases the response to the output gap. In contrast to OFI, in OFII the interest rate takes into account the next-period expectation of the interest rate as well, and it also responds to the third lag of it.[5]

5. Analysis

Due to the expected interest rate term in Equation (9) we cannot compare the two optimal rules directly. One possibility is to compare the minimum state variables (MSV) solution of the two OFs models. However, finding a parametric closed-form

[5] Of course, the expectation term on the next-period interest rate implies that the infinite stream of expected interest rates enter in Equation (9).

solution is not feasible.[6] Hence, in order to compare the two specifications we assume that the weights of the interest rate terms in both specifications of the objective functions are the same ($\lambda_i = \lambda_{\Delta i}$) and we analyze the impulse response functions (IRFs) of each specification. Specifically, we use the baseline calibration following Woodford (2003) (Appendix C). In Section 6 we show sensitivity analysis with respect to the weights of the interest rate in OFI and OFII. A further analysis is based on a decomposition of each of the IRFs to its structural components, as they appear in the model equations: The inflation rate is decomposed according to the NKPC; the output gap is decomposed according to the dynamic IS equation; and the interest rate is decomposed according to its derived optimal instrument rule, which follows from the first-order conditions of the central bank's optimization problem. The latter decomposition provides additional insights.

Figure 2 presents the IRFs relating to a one positive standard deviation of a white-noise demand shock: in the basic canonical model (without an interest rate term in the objective function) and in the two specifications of the objective function tested here.

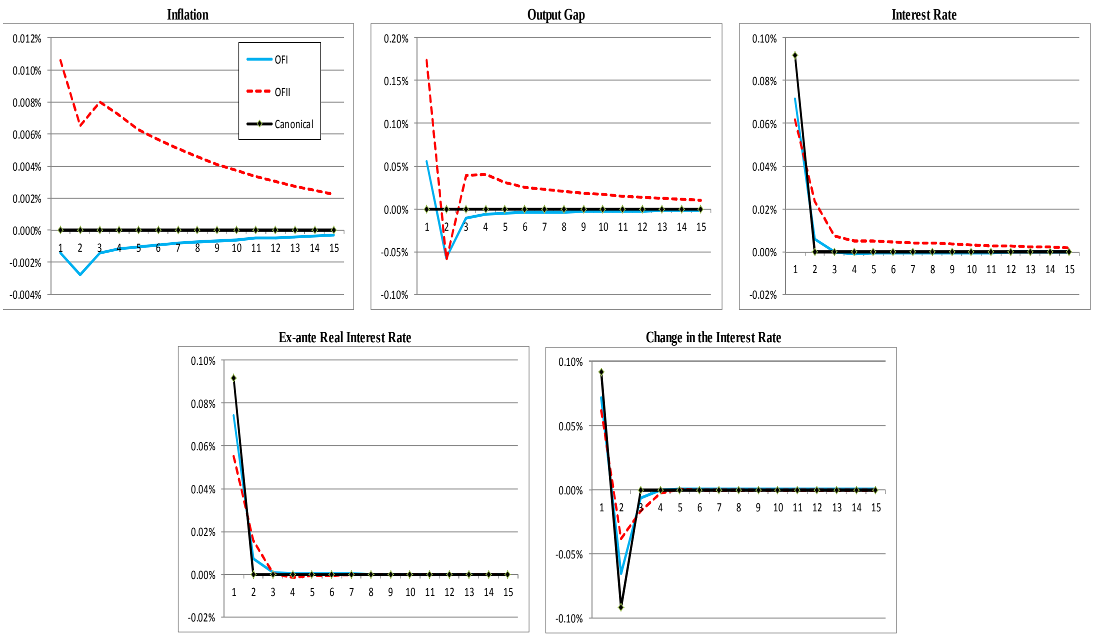

Fig. 2. Impulse response function relating to a positive one standard deviation white-noise demand shock under three OFs: Objective Function I (solid line); Objective Function II (dashed line); and the Basic canonical Objective Function (diamond line).

[6] In deriving the solution of optimal policy, a necessary step is to reorder the Schur decomposition (Söderlind 1999). This step requires numerical values of the eigenvalues and cannot be done based on parameters only.

5.1. The basic canonical objective function

As noted by Giannoni (2012), in the basic objective function (without an interest rate term, Appendix A.1) the demand shock is fully sterilized by the interest rate (Figure 2, stared line) and hence both inflation (top left) and output gap (top center) remain unchanged.

5.2. Objective function I

In contrast to the basic objective function, in OFI the demand shock seeps into the output gap and inflation. This stems from the interest rate term in the objective function, which causes a tradeoff between stabilizing inflation and the output gap and stabilizing the interest rate. The main counter-intuitive response is the *negative* inflation after the positive demand shock.[7]

Following a white-noise[8] positive demand shock the interest rate is raised in order to offset the shock (Figure 2, solid line). However, the interest rate term in the objective function prevents full sterilization of the shock, leading to a more accommodating monetary policy than in the basic objective function (Figure 2, top right), as reflected by the lower ex-ante real interest rate ($i_t - E_t\{\pi_{t+1}\}$) (bottom left) in the first period, leading to an increase in the output gap.

Figure 3 depicts the relative monetary policy stance in OFI compared to the basic case, computed by the gap between the real ex-ante interest rate in the two specifications. When the real interest rate in OFI is higher than in the basic case, monetary policy under OFI is relatively contracting, as reflected by a negative value in Figure 3, and vice versa. From the second period onward, the output gap in OFI turns negative and converges to steady state from below. That is, the negative output gap is explained by the relatively contracting monetary policy under OFI compared to the basic objective function.

At this point we should explain why the interest rate in the second period in OFI does not return to steady state despite a “fine” in the objective function to such a deviation. Recall the super-inertial characteristic of the OFI policy (Section 3). When the demand shock hits the economy, the interest rate is lower under OFI than in the basic model due to the interest rate term in the objective function. However, from the second period onward the interest rate responds in a super-inertial—that is,

[7] Giannoni (2012, Figure 2) shows a similar result with a minor positive inflation in the first period.
[8] The IRFs under an AR(1) process of the demand shock portray a similar picture.

aggressive—way to the lagged interest rate, which was raised to offset the expansionary output gap in the first period. This response is absent in the basic model.

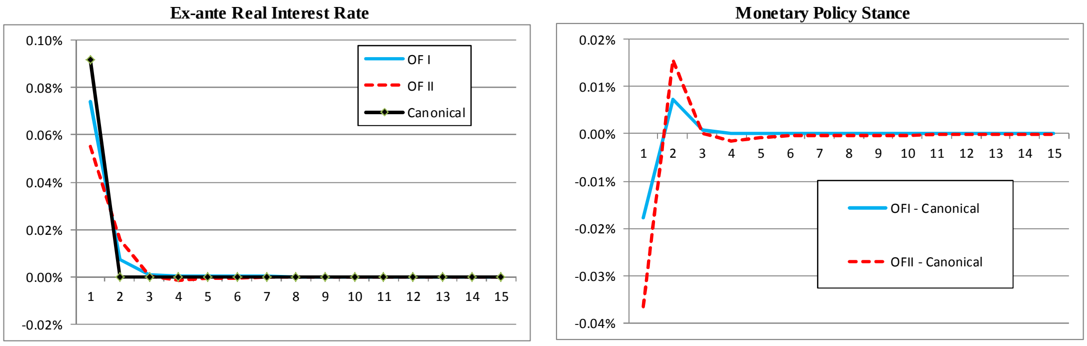

Fig. 3. Impulse response function of real ex-ante interest rate (top panel) relating to a positive one standard deviation white-noise demand shock under three OFs: Objective Function I (solid line); Objective Function II (dashed line); and the Basic Objective Function (diamond line). The monetary policy stance (right panel) is reflected by the gap between the real ex-ante interest rate under each of the two tested objective functions and the real ex-ante interest rate under the basic objective function.

Figure 4 presents the decomposition of the IRFs of inflation, output gap and interest rate according to the Phillips curve, the dynamic IS equation and the interest rate rule, respectively. The inflation IRF decomposition (Figure 4, top right) shows that the positive output gap in the first period is more than offset by the expected negative inflation due to the negative output gap from the second period onward (Figure 4, bottom), as was explained above.

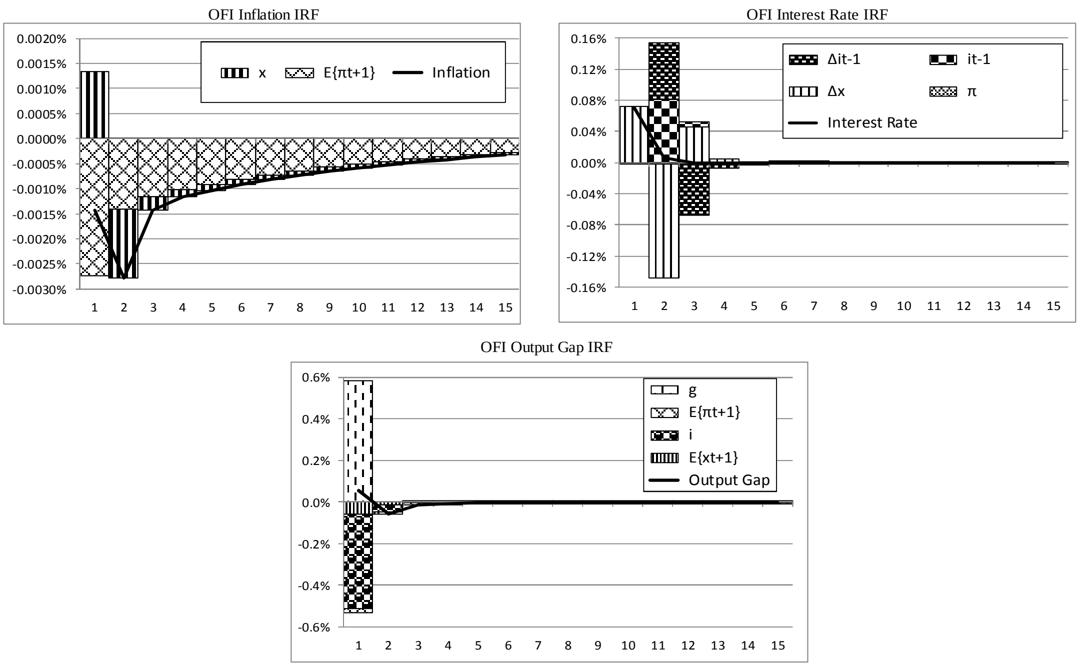

Fig. 4. Structural decomposition of OFI demand shock IRFs. Each IRF is decomposed into the structural components, as they appear in the model equations: the interest rate IRF is decomposed according to its derived instrument rule (top left); the inflation IRF is decomposed according to the NKPC (top right); and the output gap IRF is decomposed according to the dynamic IS equation (bottom).

To see this, iterating the NKPC Equation (3) forward and using the law of iterated expectations, we get:

$$\pi_t = E_t \sum_{j=0}^{\infty} \beta^j \, [\kappa x_{t+j}]. \tag{10}$$

Iterating (10) one period ahead and taking expectations, we get that the one period ahead expected inflation is a function of the future path of the expected output gap:

$$E_t\{\pi_{t+1}\} = \kappa E_t \sum_{j=0}^{\infty} \beta^j \, [x_{t+1+j}]. \tag{11}$$

5.3. Objective function II

As in OFI, in OFII the demand shock also seeps into the output gap and inflation. However, the first difference interest rate term in OFII yields, as expected, a smoother interest rate path in comparison to OFI, as shown in Figure 2. The results from the OFII specification are both positive output gaps and inflation rates after a positive demand shock, in contrast with the negative response of the inflation rate in OFI. An exception is the output gap in the second period, for which both specifications yield negative effects.

The interest rate first-difference term in OFII reflects a “fine” on interest rate *changes* rather than on interest rate *levels* under OFI. Hence, OFII yields a smother path of the interest rate in comparison to OFI and to the basic case. The smoothed path of interest rates implies an expansionary monetary policy (excluding the second period) as reflected in the negative gaps between the real interest rate (ex-ante) in OFII and its counterparts in OFI and in the basic case (Figure 3). As monetary policy is relatively expanding, the output gap is positive in all periods (but the second), and hence inflation is also positive, as explained above. This transmission mechanism from the interest rate to the output gap and inflation is also shown in Figure 5, which presents the decomposition of the IRFs under OFII. The relatively expansionary policy in OFII leads to positive expected output gap (excluding the second period, Figure 5, bottom) and in turn to positive expected inflation (Figure 5, top right).

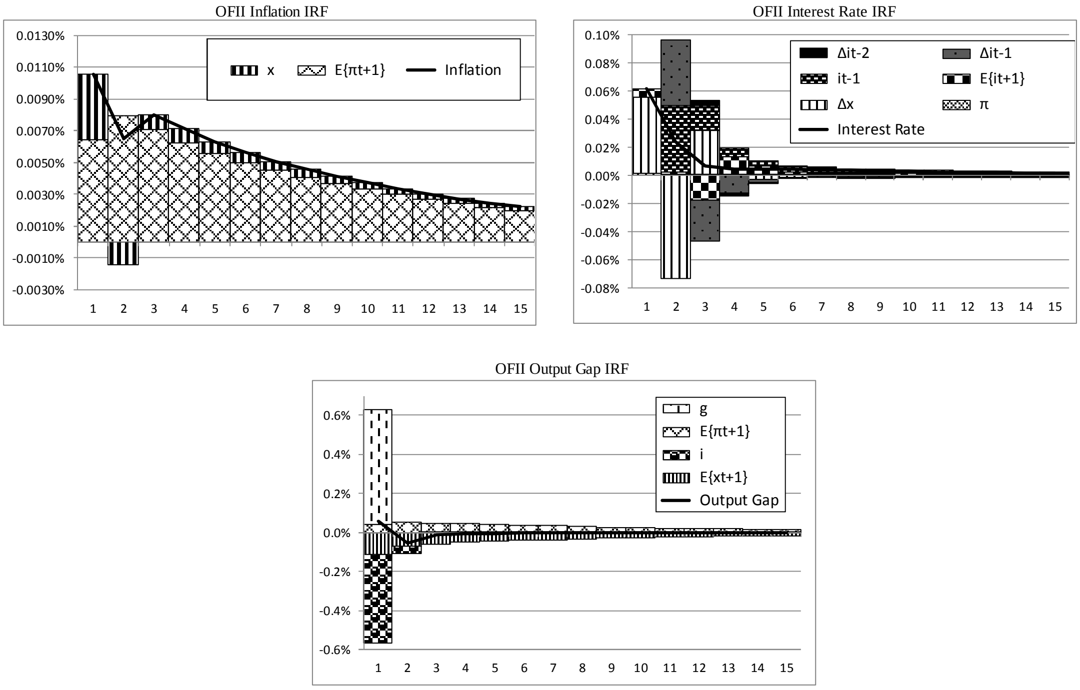

Fig. 5. Structural decomposition of OFII demand shock IRFs. Each IRF is decomposed into the structural components, as they appear in the model equations: the interest rate IRF is decomposed according to its derived instrument rule (top left); the inflation IRF is decomposed according to the NKPC (top right); the output gap IRF is decomposed according to the dynamic IS equation (bottom).

Table 1 summarizes the variances of inflation, output gap and interest rate under each of the three objective functions in the presence of demand shocks, assuming the same coefficient of the interest rate term in OFI and OFII. The variance of the interest rate in OFII is lower than in OFI, and the latter is lower than in the basic case.

Table 1

Variances of target variables among different specifications of the objective function,[9] $\boldsymbol{\lambda_i = \lambda_{\Delta i} = 0.236}$.

	Var(inflation)	Var(output gap)	Var(interest rate)
Canonical: $L_t = \pi_t^2 + \lambda_x x_t^2$	0	0	$0.0859^2 = (\sigma^{g}/\sigma)^2$
OFI: $L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2$	0.0040^2	0.0765^2	0.0669^2
OFII: $L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_{\Delta i} (\Delta i_t)^2$	0.0203^2	0.1895^2	0.0632^2

As may be expected, a lower volatility of the interest rate reflects a more restricted ability of monetary policy to offset the demand shocks. Hence, the variances of the output gap and inflation are higher under OFII than under OFI. Under OFII, the

[9] The variances were computed using 100,000 Monte-Carlo simulations of each specification.

standard deviation of inflation is five times higher, and the standard deviation of the output gap is 2.5 times higher.

6. Sensitivity Analysis

We showed that under a benchmark calibration of the Basic New Keynesian model, for the same weights in OFI and OFII, under OFI, (i_t^2), the response to a positive demand shock leads to a negative inflation rate and output gap (excluding the output gap in the first period) which converge to the steady state from below. In contrast, under OFII, $(\Delta i_t)^2$, inflation and output gap are positive and converge to steady state from above. We also showed that the variance of the interest rate in OFII is lower than in OFI, while the variances of the output gap and inflation are higher.

In this section we test whether these results are robust to a range of structural parameters. Specifically, we conduct a sensitivity analysis of the inflation, output gap and interest rate responses with respect to the (inverse) risk aversion parameter (σ), the price stickiness (θ) and the ratio of output gap to interest rate term weights in the objective function. We choose to test sensitivity with respect to σ and θ as these parameters affect the transmission from the demand shock to the economy through the structural slope of the NKPC, κ (Equation 5), and the dynamic IS Equation (4).[10]

The literature reports a wide range of the relative weights in the objective functions and σ (Table 2).[11] Hence, we conduct sensitivity analysis for $\rho_l = \lambda_{i\,/\,\Delta i}/\lambda_x = \{\frac{1}{6}, \frac{2}{3}, 1, 1\frac{1}{2}, 6\}$ and $\sigma = \{1, 2, 4, 8\}$. As for θ, we let it vary according to Table 3:

Table 2

Published estimated weights in the objective function and (inverse) risk aversion parameter

	λ_x	λ_i	$\lambda_{\Delta i}$	σ
Woodford (2003)	0.048	0.236/0.077/0.277		1/0.1571
Givens (2012)	0.0401		0.6309	1.3667
Adolfson et al. (2011)	1.091		0.476	5

[10] A sensitivity test with respect to β=[0.98,0.99,0.999] showed almost no effect on the results.

[11] Table 2 reports estimates assuming timeless perspective framework, as these estimates are sensitive to both the monetary policy framework and to the model of the economy.

Table 3

Calvo parameter and expected duration of price stickiness (quarters)

θ	0.01	0.5	2/3	0.75	5/6
Price stickiness	~1	2	3	4	5

6.1. Sensitivity analysis –objective function I

Figure 6 shows the sensitivity of the first period responses of the interest rate, output gap and inflation to a positive one standard deviation white-noise demand shock under OFI to the values of $\rho_l = \lambda_i/\lambda_x$ and σ in the range stated above, and for θ=2/3.[12] For example, the dark blue surface in the interest rate figure (left) presents the combinations of ρ_l and σ for which the interest rate rises by less than 0.005% (according to the color bar) in the first period as a response to the positive demand shock. Figure 6 reveals that the result of a *negative* inflation in the first period after a positive demand shock under OFI is robust (Figure 6, right).

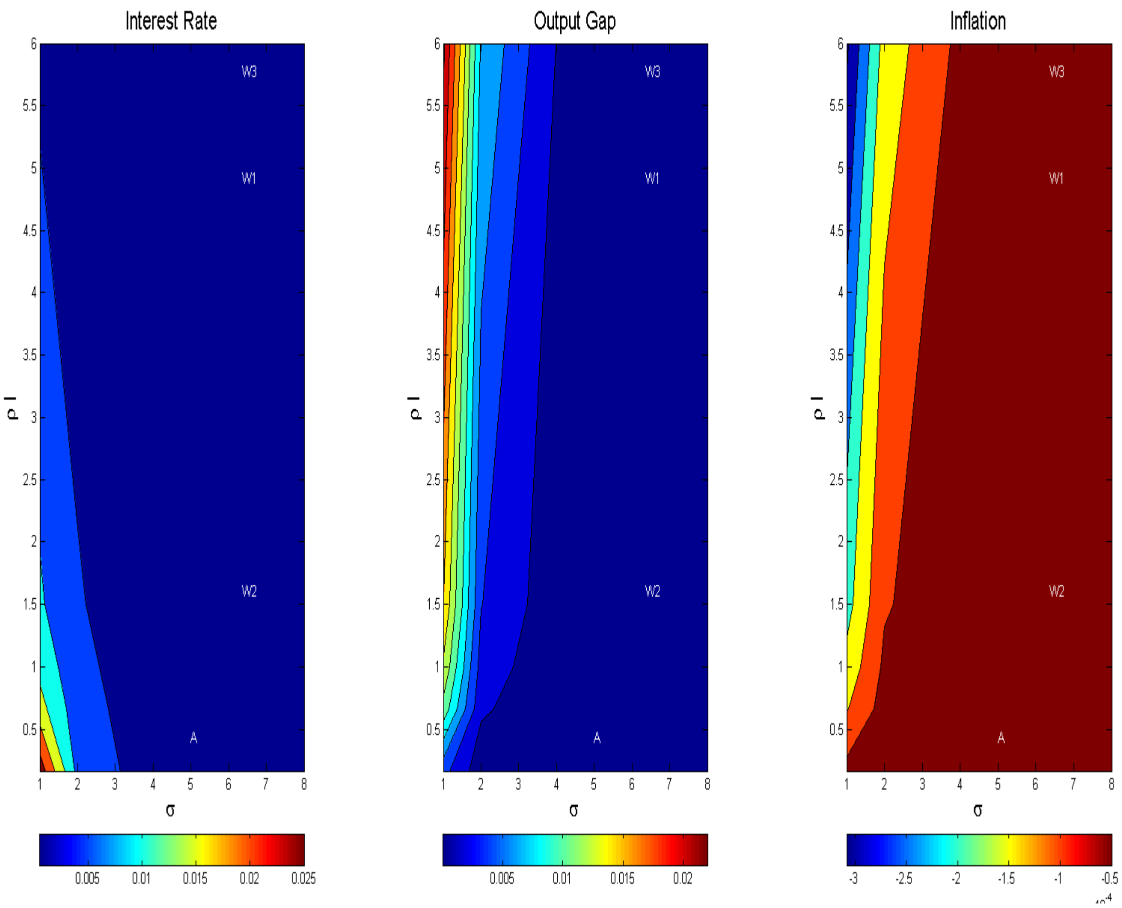

Fig. 6. First period response of interest rate (left), output gap (center) and inflation (right) to a positive one standard deviation white-noise demand shock under OFI for different values of $\rho_l = \lambda_i/\lambda_x$ and σ. "Wi" denotes Woodford's calibrations and "A" denotes Adolfson et al.'s estimation (Table 2). θ=2/3.

[12] As θ increases, it mainly affects output gap.

Figure 7 duplicates Figure 6 for the average of the responses in the first five periods, and shows that the results obtained under Woodford's benchmark calibration (Appendix C) are robust under various calibrations - a *negative* inflation after a positive demand shock under OFI. The main difference between the first period impulse response function (Figure 6) and that of the first five periods' average (Figure 7) is that the output gap becomes negative from the second period onward and thus leads to the negative inflation responses (Subsection 5.2).

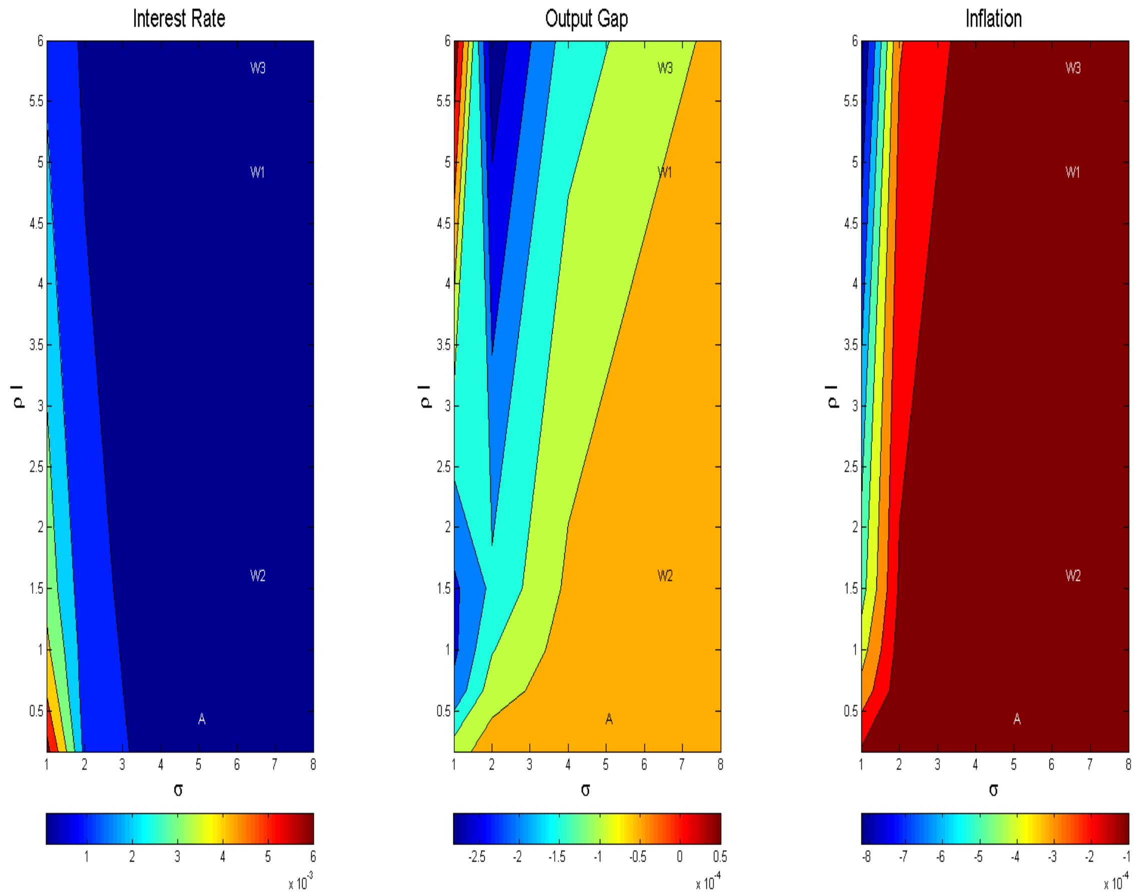

Fig. 7. The average first five periods' response of interest rate (left), output gap (center) and inflation (right) to a positive one standard deviation white-noise demand shock under OFI for different values of $\rho_l = \lambda_i/\lambda_x$ and σ. "Wi" denotes Woodford's calibrations and "A" denotes Adolfson et al.'s estimation (Table 2). θ=2/3.

Figures 6 and 7 show that as ρ_l and σ decline, the nominal interest rate responds more aggressively (left), leading to a lower negative output gap (center of Figure 7, from the second period onward) and hence to a lower negative inflation rate (right). The reason for these results is that: I) A lower ρ_l implies a smaller relative "fine" on deviations of the interest rate term (in both OFI and OFII) to output gap's deviations—that is, a smaller tradeoff between the interest rate and inflation and the output gap. Hence the interest rate is less constrained. II) A lower σ implies a higher slope of the NKPC which yields a higher influence of the output gap on the inflation, as from Equation (5) $\partial\kappa/\partial\sigma = -\frac{(1-\theta)(1-\beta\theta)}{\theta\sigma^2(1+\epsilon\omega)} < 0$. The higher slope also yields, *ceteris paribus*, a higher relative weight of the output gap/ lower relative "fine" on the interest rate in the objective function (Equation (6)). Hence, as ρ_l and σ decline, the interest rate's influence raises. Note, that although the output gap response is not monotonic (Figure 7, center) it remains negative, leading to negative inflation.

As for θ, as $\partial\kappa/\partial\theta = \frac{(\omega+1/\sigma)(1-\beta\theta^2-1)}{\theta^2(1+\epsilon\omega)} < 0$, when θ increases—that is, when prices become stickier—the policy influence is weaker, following the logic stated above (Appendix D duplicates Figures 6 and 7 for θ=5/6).

To summarize, Figures 6 and 7 show that under OFI, after a positive white-noise demand shock, the optimal policy from a timeless perspective leads to a counter-intuitive *negative* inflation rate and output gap (from the second period onward) for a wide range of parameter values.

6.2. Sensitivity analysis –objective function II

Figures E.1 and E.2 (Appendix E) duplicate Figures 6 and 7 under OFII, and show that the *positive* inflation and output gap responses after a positive demand shock reported in Section 5 are also robust.

The mechanism described in Subsection 6.1 also explains the higher inflation under OFII, as ρ_l and σ decline. Similar to OFI, in OFII as ρ_l and σ decline the nominal interest rate responds more aggressively (left), but in contrast to OFI, output gap and inflation rate increase.

To summarize, as ρ_l and σ decline, the deviation between the inflation response in OFI compared to OFII increases.

7. Designing Monetary Policy—an Augmented Taylor Rule

We showed that the negative inflation after a positive demand shock under OFI and the positive inflation under OFII are robust (Section 6). Based on the analysis and the comparison between the two targeting rules, Equations (7) and (9), we argue that the difference between the inflation's response under the two tested objective functions stems from the different inertia of the interest rate in the targeting rules.

To test this hypothesis, we assume that the interest rate is set according to an augmented Taylor rule:

$$i_t = \rho_1 i_{t-1} + \rho_2 i_{t-2} + |[1 - (\rho_1 + \rho_2)]|(\tau_\pi \pi_t + \tau_x \Delta x_t). \qquad (12)$$

The augmentation consists of (1) the second lag of the interest rate; and (2) of the absolute value attached to the $1 - (\rho_1 + \rho_2)$ term. Note that the OFI targeting rule (Equation (7)) is a special case of Equation (12). Under OFI, both $1 - (\rho_1 + \rho_2)$ and the coefficients of the target variables are negative, $\tau_\pi = -\beta/\lambda_i$; $\tau_x = -(\beta\lambda_x)/(\kappa\lambda_i)$. Hence the representation of Equation (7) in terms of Equation (12) is given by:

$$\rho_1 = 1 + (\sigma\kappa + 1)/\beta;\ \rho_2 = -\beta^{-1}; 1 - (\rho_1 + \rho_2) = -\sigma\kappa/\beta; \qquad (13)$$
$$\tau_\pi = \beta/\lambda_i;\ \tau_x = (\beta\lambda_x)/(\kappa\lambda_i).$$

To test our hypothesis, we test the endogenous variables, and specifically the inflation response, to a positive white-noise demand shock where we let ρ_1 vary in the range of $\rho_1 = \{0.61, 0.81, 1.01, \dots, 3.01\}$[13] and $\frac{\rho_2}{\rho_1} = \{-1, -0.8, -0.6, \dots, 1\}$. The lower bound of ρ_1 is set in accordance with the lower empirical bound. The $\frac{\rho_2}{\rho_1}$ bounds are set in order for ρ_2 to only mildly offset ρ_1; note that ρ_2 can be negative. Finally, we calibrate $\tau_\pi = 1.5$ and $\tau_x = 0.5$ as in the canonical Taylor (1993) rule.

Figure 8 verifies our hypothesis—there is a frontier which separates between a positive and negative average inflation response under Equation (12). The frontier is where $\rho_1 + \rho_2 = 1$ (the white starred line); for $\rho_1 + \rho_2 < 1$ the inflation response is positive, in accordance with the standard intuitive response, while for $\rho_1 + \rho_2 > 1$ it is negative.[14] The frontier is sketched in the range of $\rho_1 < 2.25$, because 2.25 is the maximum value of ρ_1 under the OFI specification in the sensitivity analysis (Section 6). Hence, the augmented Taylor-type rule in Equation (12) can lead to either a positive or a negative inflation response to a positive demand shock, depending on the sum of the coefficients of the lagged interest rates, the interest rate inertia.

[13] When $\rho_1 + \rho_2 = 1$, Equation (12) leads to indeterminacy.
[14] Similar results are obtained for $\tau_\pi = 1.5$ and $\tau_x = 1$ or for the baseline calibration of OFI— τ_π =4.1658 and $\tau_x = 8.4585$.

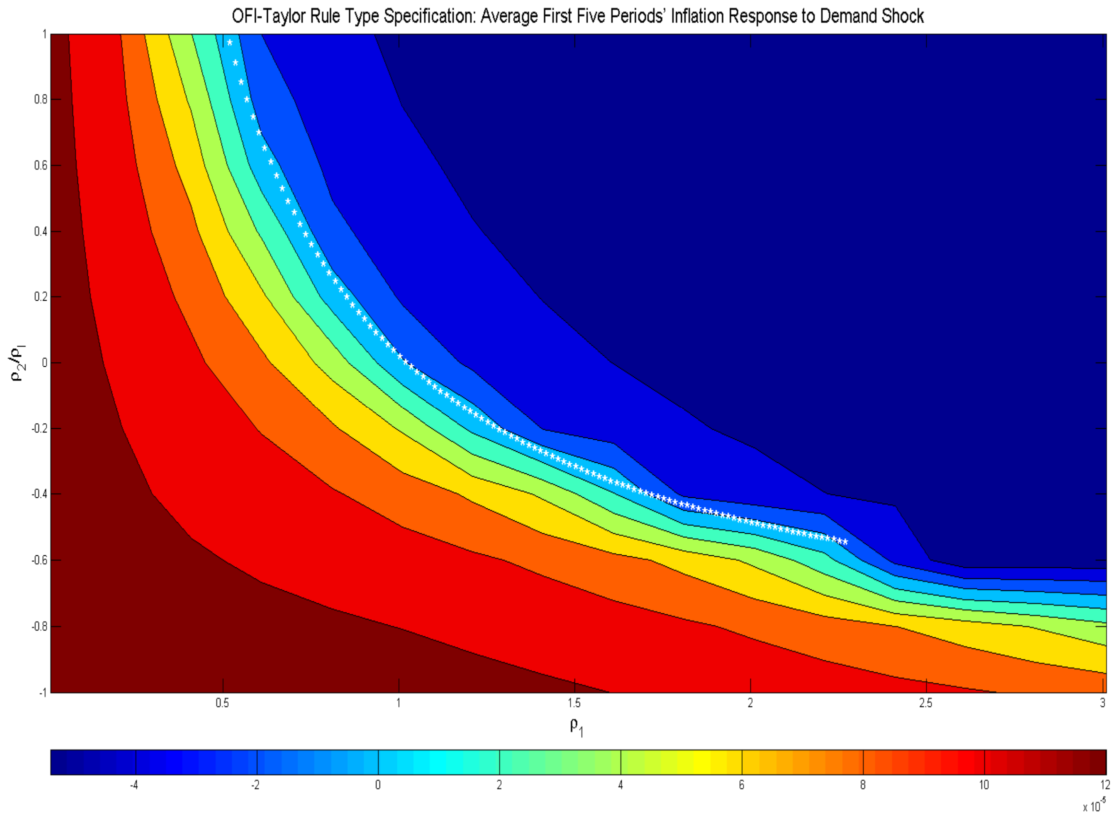

Fig. 8. The average first five periods' response of inflation to a positive one standard deviation white-noise demand shock under $i_t = \rho_1 i_{t-1} + \rho_2 i_{t-2} + |[1-(\rho_1+\rho_2)]|(\tau_\pi \pi_t + \tau_x \Delta x_t)$ for different values of ρ_1 and ρ_2. The (white) starred line presents the $\rho_1 + \rho_2 = 1$ line—the "frontier".

Figure 9 illustrates our results and maps the different policy rules in the ρ_1 and $\frac{\rho_2}{\rho_1}$ region.

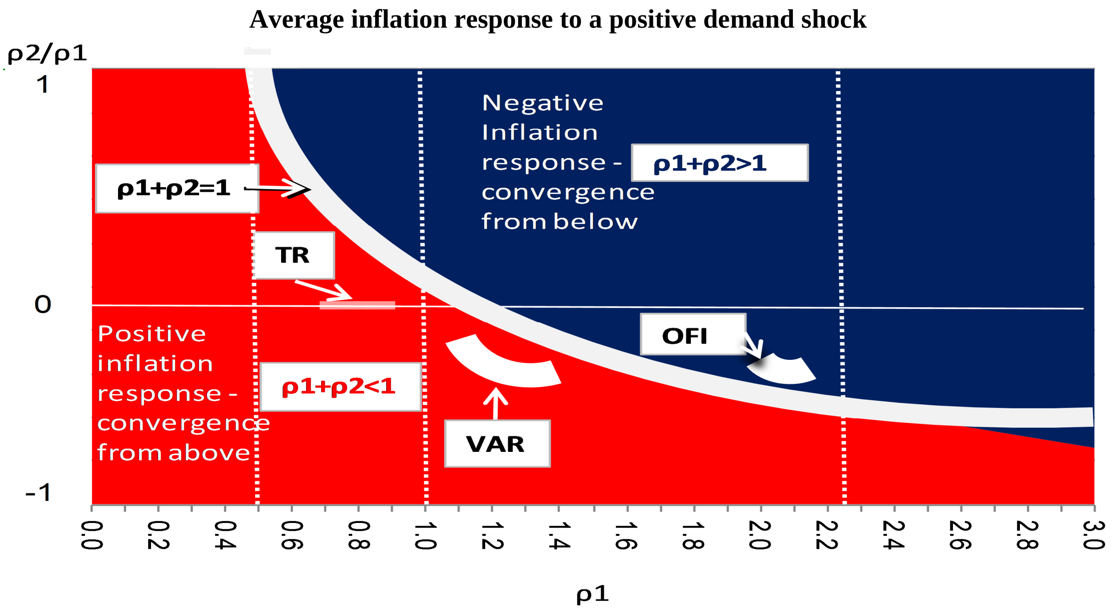

Fig. 9. The average first five periods' response of inflation to a positive one standard deviation white-noise demand shock under $i_t = \rho_1 i_{t-1} + \rho_2 i_{t-2} + |[1-(\rho_1+\rho_2)]|(\tau_\pi \pi_t + \tau_x \Delta x_t)$ for different values of ρ_1 and ρ_2.

The standard Taylor-type rules—where $0.6 < \rho_1 < 0.9$ and $\rho_2 = 0$—lead to a positive inflation response (the TR block in the figure). Similarly, Coibion and Gorodnichenko (2012) as well as some empirical VAR systems with two lags find that $\rho_1 > 1$, $\rho_2 < 0$ and $\rho_1 + \rho_2 < 1$ (the VAR block in the figure). The (theoretical) OFI area is in the negative inflation response, as was demonstrated in the sensitivity analysis, because $\rho_1 + \rho_2 = 1 + \sigma\kappa/\beta$.

8. Summary

We compared two broadly used specifications of objective functions for the monetary authority with respect to the interest rate; a squared interest rate term (OFI) and a squared *first difference* of the interest rate (OFII). The main difference between the tested objective functions is in the inflation response to a positive demand shock–negative under OFI and positive under OFII. The reason for the negative inflation dynamics under OFI is the strong (denoted in the literature as super-inertial) response of the interest rate to its lags, which leads to a contractionary monetary policy, compared with a more smoothed response derived under OFII, which leads to an expansionary monetary policy.

Based on the comparison between the targeting rules under OFI and OFII, we show that an augmented Taylor-type rule with two lags of the interest rate can lead to different inflation responses to a positive demand shock. It leads to a positive inflation response when the sum of the coefficients of the lagged interest rate (the "frontier") is below one, while it leads to a negative average inflation response when the "frontier" is above one. Hence, setting the "frontier" to be above one, where it is plausible depending on the lower bound of the interest rate—can be useful in an environment of inflation that is too low due to negative demand shocks—a conventional tool which may be used to get unconventional results. The main challenge is how to implement such a rule and to communicate and deliver the chosen policy to the public, as the interest rate paths are similar.

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Appendix A - Derivation of the first-order conditions under OFI

The Lagrangian of the central bank's problem under Objective Function I (OFI hereafter) is given by (Woodford (2003)):

$$W_t = E\{\sum_{t=0}^{\infty}\beta^t\{0.5[\pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^{\ 2}] + \varphi_{1,t}[x_t - x_{t+1} + \sigma i_t - \sigma\pi_{t+1}] + \varphi_{2,t}[\pi_t - kx_t - \beta\pi_{t+1}]\}, \qquad \text{A.1.}$$

where $\phi_{1,t}$ and $\phi_{2,t}$ are the Lagrange multipliers of the dynamic IS Equation (4) and the Phillips curve (3), respectively.

The first-order conditions under the timeless perspective policy are given by:

$$\partial W_t / \partial \pi_t = \pi_t - \sigma\varphi_{1,t-1}/\beta + \varphi_{2,t} - \varphi_{2,t-1} = 0 \ , \qquad \text{A.2.}$$

$$\partial W_t / \partial x_t = \lambda_x x_t + \varphi_{1,t} - \varphi_{1,t-1}/\beta - \kappa\varphi_{2,t} = 0 \ , \qquad \text{A.3.}$$

$$\partial W_t / \partial i_t = \lambda_i i_t + \sigma\varphi_{1,t} = 0 \ . \qquad \text{A.4.}$$

These conditions apply $\forall t$ in a Timeless Perspective (TP) commitment policy (Woodford 2003 p. 523). The TP framework assumes that the CB had committed far in the past, and consequently it has both a direct influence on expectations and a time consistent policy. One interpretation of the TP framework is that it may reflect policy of central banks which gained credibility. Adolfson, et al. (2008) use the TP framework to describe the Riksbank policy, and Minford and Ou (2013) find that the

TP framework fits Federal Reserve policy in the US better than a Taylor rule with interest rate smoothing.

Extracting the interest rate under OFI from the first-order conditions (Giannoni (2012)) yields the optimal OFI interest rate rule, Equation (7) in the main text:

$$i_t = (\sigma\kappa/\lambda_i)\pi_t + (\sigma\lambda_x/\lambda_i)\,\Delta\, x_t + (1 + \sigma\kappa/\beta)i_{t-1} + \beta^{-1}\,\Delta\, i_{t-1}\;, \tag{7}$$

where $\Delta i_{t-1} \equiv i_{t-1} - i_{t-2}$.

A.1. Special case - the canonical objective function

The multiplier $\phi_{1,t}$ represents the marginal (welfare) loss due to a dynamic IS equation shift, and $\phi_{2,t}$ represents the marginal (welfare) loss due to a Phillips curve shift.

For the following discussion it is beneficial to treat the canonical objective function as a benchmark. In the canonical objective function there is no interest rate term—that is, $\lambda_{\text{i}} \equiv 0$. Hence, the first-order condition A.4 yields $\phi_{1,t} = 0\ \forall t$, implying that in this case the dynamic IS equation is irrelevant for the optimization problem of the central bank (see, e.g., Giannoni and Woodford, 2005, footnote 6 and McCallum and Nelson, 2004). In contrast, in OFI a tradeoff emerges between stabilizing the interest rate and stabilizing inflation and the output gap when a demand shock hits the economy. The dynamic IS equation (4) in this case is relevant for the optimization problem of the central bank, reflected by $\phi_{1,t}$ being different from zero.

Returning to the canonical objective function case, the first-order conditions A.2 and A.3 become:

$$\pi_t + \phi_{2,t} - \phi_{2,t-1} = 0, \qquad \text{A.5.}$$

$$\lambda_x x_t - \kappa\phi_{2,t} = 0. \qquad \text{A.6.}$$

These two first-order conditions yield the familiar targeting rule of the timeless perspective (TP) policy which was proposed by Woodford (2003a), under the canonical objective function:

$$\pi_t = (\lambda_x/\kappa)(x_{t-1} - x_t) \Leftrightarrow \pi_t = -(\lambda_x/\kappa)\,\Delta\, x_t. \qquad \text{A.7.}$$

Under the canonical objective function specification, optimal inflation is a negative function of the *change* in the output gap. This result was highlighted by Woodford (1999) as “optimal monetary policy inertia” regardless of any inertia in the structural shocks, and is consistent with the “speed limit” policy advocated by Walsh (2003). The targeting rule A.7 stems from the commitment of the central bank under the timeless perspective policy, which binds its action in each date to its previous actions as reflected in the lagged output gap.

Extracting the interest rate from the dynamic IS equation (4) and using the Phillips curve (3) and the targeting rule A.7, the interest rate rule is given by:

$$i_t = \frac{1-\kappa/\lambda_x}{\beta\sigma[\pi_t - x_t]} + \sigma^{-1} g_t. \qquad \text{A.8.}$$

Hence, in the canonical objective function the interest rate responds to the same period inflation, output gap and shocks. Furthermore, the interest rate sterilizes the demand shock, rising by $\sigma^{-1} g_t$.

Appendix B–Derivation of the Optimal Interest Rate Rule under OFII

The Lagrangian the CB is facing is given by:

$$\begin{aligned} W = E_t \sum_{t=0}^{\infty} \beta^t \{ & 0.5[\pi_t^2 + \lambda_x x_t^2 + \lambda_{\Delta i}(\Delta i_t)^2] + \varphi_{1,t}[x_t - x_{t+1} + \sigma i_t - \sigma \pi_{t+1}] \\ & + \varphi_{2,t}[\pi_t - k x_t - \beta \pi_{t+1}]\} \end{aligned} \qquad \text{B.1.}$$

The first-order conditions are given by:

$$\partial W / \partial \pi_t = \pi_t - \sigma \varphi_{1,t-1} / \beta + \varphi_{2,t} - \varphi_{2,t-1} = 0 \qquad \text{B.2.}$$

$$\partial W / \partial x_t = \lambda_x x_t + \varphi_{1,t} - \varphi_{1,t-1} / \beta - \kappa \varphi_{2,t} = 0 \qquad \text{B.3.}$$

$$\partial W / \partial i_t = \lambda_{\Delta i} \Delta i_t - \beta \lambda_{\Delta i} E_t\{\Delta i_{t+1}\} + \sigma \varphi_{1,t} = 0 \qquad \text{B.4.}$$

From A.4, $\varphi_{1,t}$ is given by:

$$\varphi_{1,t} = \frac{\beta \lambda_{\Delta i} E_t\{\Delta i_{t+1}\}}{\sigma} - \frac{\lambda_{\Delta i} \Delta i_t}{\sigma} \qquad \text{B.5.}$$

Plugging B.5 into B.3, multiplying by β and extracting $\varphi_{2,t}$ yields:

$$\varphi_{2,t} = \frac{\lambda_x}{\kappa} x_t + \frac{\lambda_{\Delta i}}{\sigma\kappa}[\beta E_t\{\Delta i_{t+1}\} - 2\Delta i_t + \frac{1}{\beta}\Delta i_{t-1}] \qquad \text{B.6.}$$

Multiplying B.2 by β and inserting the Lagrangian coefficient terms using B.5 and B.6 yields:

$$\beta\pi_t - \beta\lambda_{\Delta i}\Delta i_t + \lambda_{\Delta i}\Delta i_{t-1} + \frac{\beta\lambda_x}{\kappa}\Delta x_t + \frac{\beta\lambda_{\Delta i}}{\sigma\kappa}[\beta E_t\{\Delta\Delta i_{t+1}\} - 2\Delta\Delta i_t + \frac{1}{\beta}\Delta\Delta i_{t-1}] = 0 \qquad \text{B.7.}$$

where $\Delta\Delta z_t \equiv \Delta(\Delta z_t) = \Delta(z_t - z_{t-1}) = z_t - z_{t-1} - (z_{t-1} - z_{t-2}) = \Delta z_t - \Delta z_{t-1}$

Using $\Delta\Delta z_{t-1}$ definition and extracting i_t from A.7 yields the optimal instrument (interest rate) rule, Equation (9) in the main text:

$$\begin{aligned} i_t &= [(\sigma\kappa)/(\lambda_{\Delta i}\Omega)]\pi_t + [(\sigma\lambda_x)/(\lambda_{\Delta i}\Omega)]\Delta x_t + (\beta/\Omega)E_t\{i_{t+1}\} + (1-\beta/\Omega)i_{t-1} \\ &\quad + [(\Omega-1)/(\beta\Omega)]\Delta i_{t-1} - (\beta\Omega)^{-1}\Delta i_{t-2} \end{aligned} \qquad (9)$$

where $\Omega \equiv \sigma\kappa + 2(1+\beta)$.

Appendix C–Calibration

The baseline calibration (Section 5) is based on Woodford (2003, p. 431) and Giannoni (2012). Table C.1 shows the values of the structural parameters in these papers.

Table C.1.
Calibrated Parameters

Parameter	Value	Description
β	0.99	Discount factor
ε	7.88	Elasticity of substitution among goods
θ	2/3	Calvo price parameter
σ	1/0.1571	Consumption inter-temporal elasticity of substitution (inverse)
ω	0.47	Elasticity of real marginal cost with respect to production
ρ_l	0.236/0.048	Relative weight of interest rate to output gap in the objective function
$sd(g)$ [15]	0.0372/σ	Standard deviation of demand shock

[15] Note that in Woodford (2003) and Giannoni (2012), the shock in the dynamic IS equation is the natural interest rate, in contrast with the demand shock in CGG (1999). While the specification of the shock as well as its interpretation differs across the models, the reduced form shocks are identical in terms of dynamics in the model; $\hat{g} \equiv \sigma^{-1} r_t^e$ in Giannoni (2012).

Appendix D–Sensitivity Analysis-Objective Function I, for θ=5/6

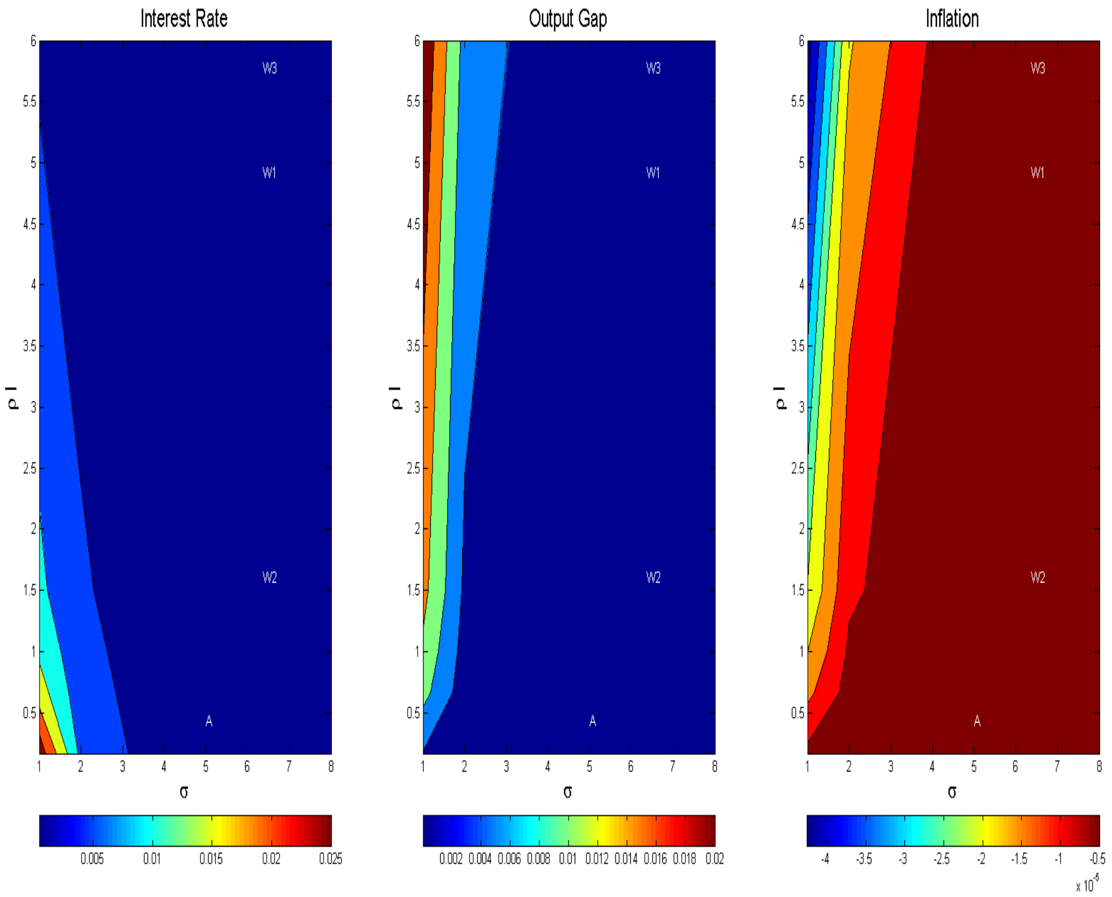

Fig. D.1. First period response of interest rate (left), output gap (center) and inflation (right) to a positive one standard deviation white-noise demand shock under OFI for different values of $\rho_l = \lambda_i/\lambda_x$ and σ. "Wi" denotes Woodford's calibrations and "A" denotes Adolfson et al.'s estimation (Table 2). θ=5/6.

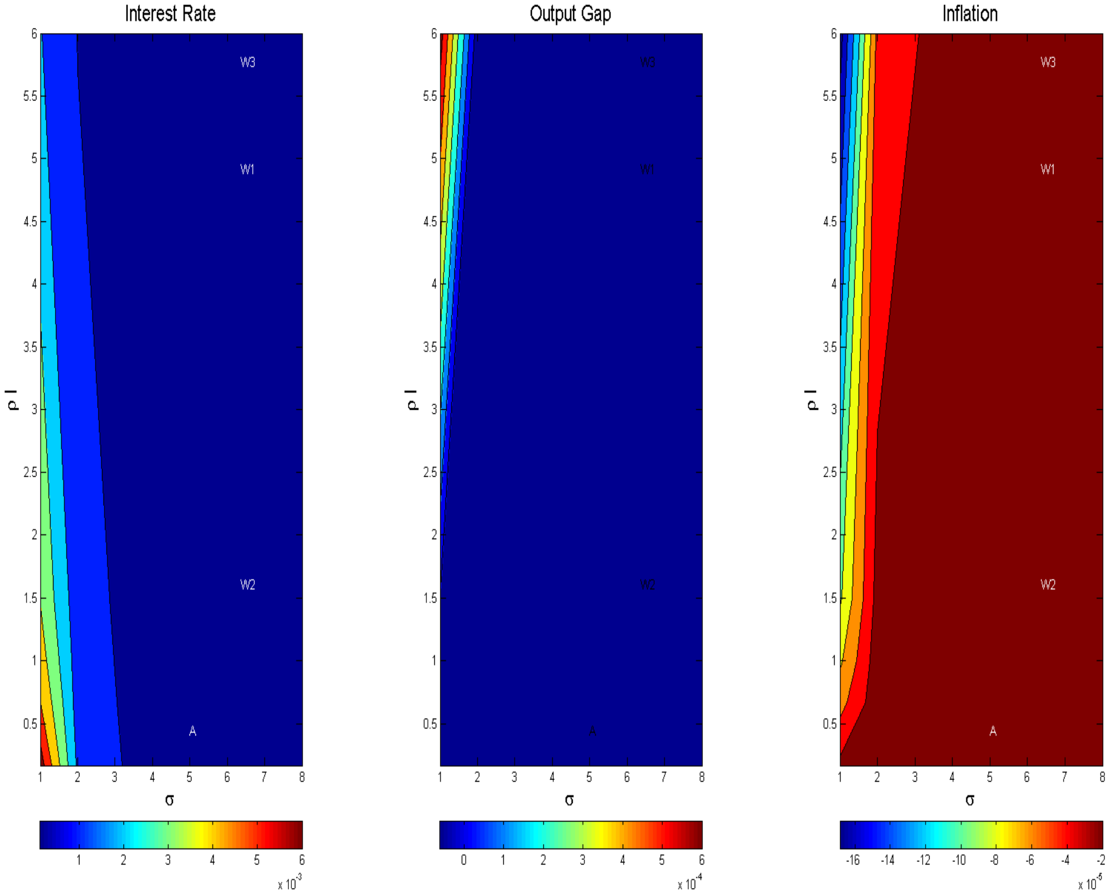

Fig. D.2. The average first five periods' response of interest rate (left), output gap (center) and inflation (right) to a positive one standard deviation white-noise demand shock under OFI for different values of $\rho_l = \lambda_i/\lambda_x$ and σ. "Wi" denotes Woodford's calibrations and "A" denotes Adolfson et al.'s estimation (Table 2). θ=5/6.

Appendix E–Sensitivity Analysis-Objective Function II, for θ=2/3

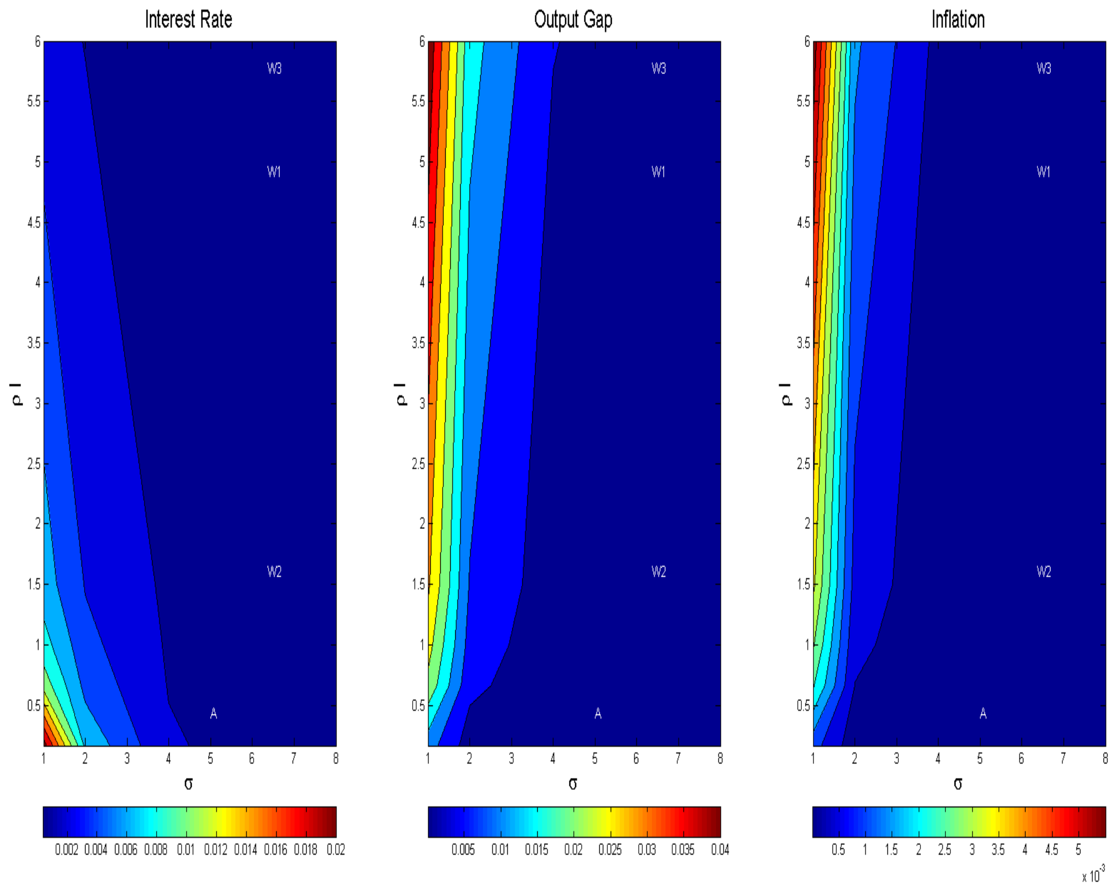

Fig. E.1 First period response of interest rate (left), output gap (center) and inflation (right) to a positive one standard deviation white-noise demand shock under OFII for different values of $\rho_l = \lambda_i/\lambda_x$ and σ**.** "Wi" denotes Woodford's calibrations and "A" denotes Adolfson et al.'s estimation (Table 2). θ=2/3.

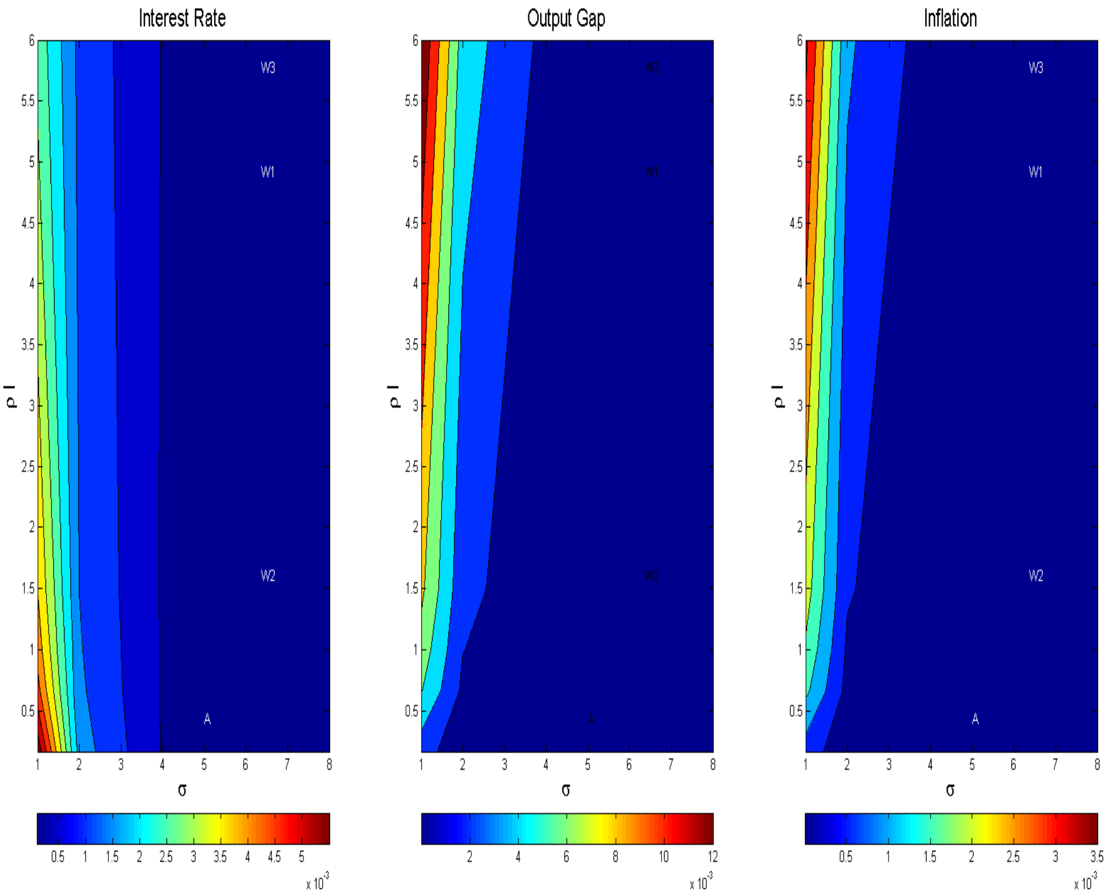

Fig. E.2 The average first five periods' response of interest rate (left), output gap (center) and inflation (right) to a positive one standard deviation white-noise demand shock under OFII for different values of $\rho_l = \lambda_i/\lambda_x$ and σ. "Wi" denotes Woodford's calibrations and "A" denotes Adolfson et al.'s estimation (Table 2). θ=2/3.